

$$\begin{array}{c}
\frac{\Delta^* \vdash \Gamma}{\Delta^*; \Gamma \vdash \mathbf{true} : (\mathbf{bool}) @ \perp} \text{WFT:TRUE} \qquad \frac{\Delta^* \vdash \Gamma}{\Delta^*; \Gamma \vdash \mathbf{false} : (\mathbf{bool}) @ \perp} \text{WFT:FALSE} \\
\\
\frac{\Delta^* \vdash \Gamma \quad x : \sigma \in \Gamma}{\Delta^*; \Gamma \vdash x : \sigma} \text{WFT:VAR} \qquad \frac{\Delta^*; \Gamma, x : \sigma_1 \vdash e : \sigma_2 \quad \Delta^* \vdash \sigma_1}{\Delta^*; \Gamma \vdash \lambda x : \sigma_1. e : \sigma_1 \xrightarrow{\perp} \sigma_2} \text{WFT:ABS} \\
\\
\frac{\Delta^*; \Gamma \vdash e_1 : \sigma_1 \xrightarrow{\ell} \sigma_2 \quad \Delta^*; \Gamma \vdash e_2 : \sigma_1}{\Delta^*; \Gamma \vdash e_1 e_2 : \sigma_2 \sqcup \ell} \text{WFT:APP} \qquad \frac{\Delta^*, \alpha : \star^\ell; \Gamma \vdash e : \sigma}{\Delta^*; \Gamma \vdash \Lambda \alpha : \star^\ell. e : \forall^\perp \alpha : \star^\ell. \sigma} \text{WFT:TABS} \\
\\
\frac{\Delta^*; \Gamma \vdash e : \forall^\ell \alpha : \star^{\ell'} . \sigma \quad \Delta^* \vdash \tau : \star^{\ell'}}{\Delta^*; \Gamma \vdash e[\tau] : \sigma[\tau/\alpha] \sqcup \ell} \text{WFT:TAPP} \qquad \frac{\Delta^*; \Gamma \vdash e_1 : \sigma_1 \quad \Delta^*; \Gamma \vdash e_2 : \sigma_2}{\Delta^*; \Gamma \vdash \langle e_1, e_2 \rangle : \sigma_1 \times^\perp \sigma_2} \text{WFT:PAIR} \\
\\
\frac{\Delta^*; \Gamma \vdash e : \sigma_1 \times^\ell \sigma_2}{\Delta^*; \Gamma \vdash \mathbf{fst} e : \sigma_1 \sqcup \ell} \text{WFT:FST} \qquad \frac{\Delta^*; \Gamma \vdash e : \sigma_1 \times^\ell \sigma_2}{\Delta^*; \Gamma \vdash \mathbf{snd} e : \sigma_2 \sqcup \ell} \text{WFT:SND} \\
\\
\frac{\Delta^*; \Gamma, x : \sigma \vdash e : \sigma \quad \Delta^* \vdash \sigma}{\Delta^*; \Gamma \vdash \mathbf{fix} x : \sigma. e : \sigma} \text{WFT:FIX} \\
\\
\frac{\Delta^*; \Gamma \vdash e_1 : (\mathbf{bool}) @ \ell \quad \Delta^*; \Gamma \vdash e_2 : \sigma \quad \Delta^*; \Gamma \vdash e_3 : \sigma}{\Delta^*; \Gamma \vdash \mathbf{if} e_1 \mathbf{then} e_2 \mathbf{else} e_3 : \sigma \sqcup \ell} \text{WFT:IF} \\
\\
\frac{\ell \sqsubseteq \ell' \quad \Delta^* \vdash \tau : \star^\ell \quad \Delta^*, \gamma : \star^\ell \vdash \sigma \quad \Delta^*; \Gamma \vdash e_{\mathbf{bool}} : \sigma[\mathbf{bool}/\gamma] \quad \Delta^*; \Gamma \vdash e_{\rightarrow} : \forall^{\ell'} \alpha : \star^\ell. \forall^{\ell'} \beta : \star^\ell. \sigma[\alpha \rightarrow \beta/\gamma] \quad \Delta^*; \Gamma \vdash e_{\times} : \forall^{\ell'} \alpha : \star^\ell. \forall^{\ell'} \beta : \star^\ell. \sigma[\alpha \times \beta/\gamma] \quad \text{where } \ell' = \mathcal{L}(\sigma[\tau/\gamma])}{\Delta^*; \Gamma \vdash \mathbf{typecase} [\gamma. \sigma] \tau e_{\mathbf{bool}} e_{\rightarrow} e_{\times} : \sigma[\tau/\gamma]} \text{WFT:TCASE} \\
\\
\frac{\Delta^*; \Gamma \vdash e : \sigma_1 \quad \Delta^* \vdash \sigma_1 \leq \sigma_2}{\Delta^*; \Gamma \vdash e : \sigma_2} \text{WFT:SUB}
\end{array}$$

Figure 2:4: Term well-formedness

$$\begin{array}{c}
\frac{\Delta^* \vdash \tau : \star^\ell}{\Delta^* \vdash (\tau) @ \perp} \text{WFTP:CON} \qquad \frac{\Delta^* \vdash \sigma_1 \quad \Delta^* \vdash \sigma_2}{\Delta^* \vdash \sigma_1 \xrightarrow{\perp} \sigma_2} \text{WFTP:ARR} \qquad \frac{\Delta^* \vdash \sigma_1 \quad \Delta^* \vdash \sigma_2}{\Delta^* \vdash \sigma_1 \times^\perp \sigma_2} \text{WFTP:PROD} \\
\\
\frac{\Delta^*, \alpha : \star^\ell \vdash \sigma}{\Delta^* \vdash \forall^\perp \alpha : \star^\ell. \sigma} \text{WFTP:ALL}
\end{array}$$

Figure 2:5: Type well-formedness